Signal and Systems

By

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كلية الهندســـة بشبرا
FACULTY OF ENGINEERING- SHOUBRA

Lecture (4)

Course Title: **Signal and Systems**

Course Code: **ELE 115**

Contact Hours: **5.**

= [2 Lect. + 2 Tut + 1 Lab]

Assessment:

Final Exam: 75%.

Midterm: ??%.

Year Work & Quizzes: 50%.

Experimental/Oral: 25%.

Textbook:

1- E. W. Kamen and B. S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd ed., Pearson Hihgher Education, 2006.
2- Benjamin C. Kuo " Automatic control systems" 9th ed., John Wiley & Sons, Inc. 2010.
3- Katsuhiko Ogata, "Modern Control Engineering", 4th Edition, 2001.

Course Description

- Introduction, fundamentals and basic properties of signals and systems, definition of open loop and closed loop systems, mathematical models of physical systems (mechanical, electrical, electromechanical systems …), control system components, block diagram simplification, signal flow graph, state variable models, Z-Transform and its properties, solving difference equations, pulse transfer function of discrete system, Fourier transforms, continuous and discrete signal analysis, transient response of first and second order control systems, real life applications such as analog and digital filters, introduction to basics of digital signal processor (DSP) and its features and capabilities of commercial applications.

Signal-Flow Graphs of State Equations

Problem: draw signal-flow graph for:

$$\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r$$

$$\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r$$

$$\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r$$

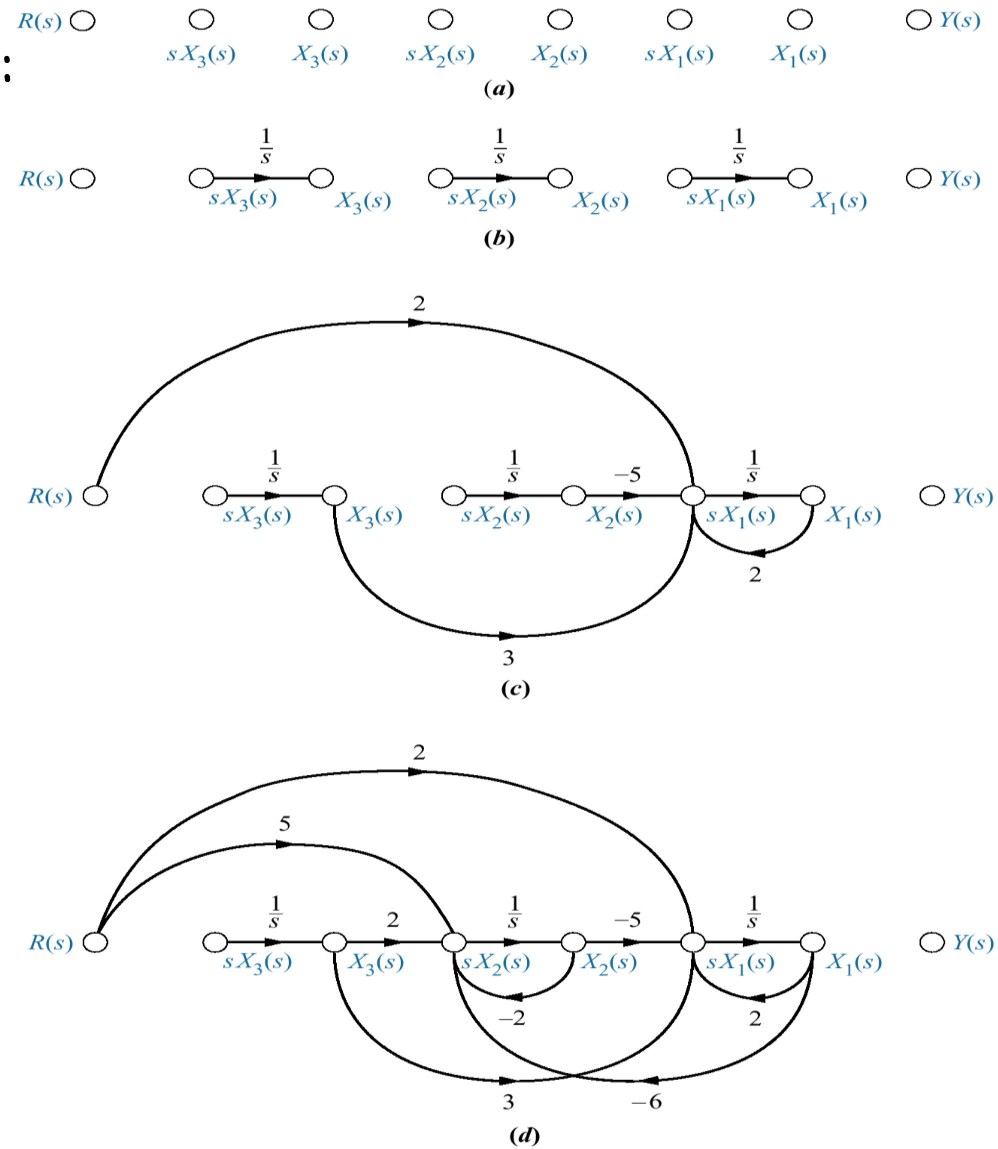
$$y = -4x_1 + 6x_2 + 9x_3$$

a. place nodes;

b. interconnect state variables and derivatives;

c. form dx1/dt ;

d. form dx2/dt

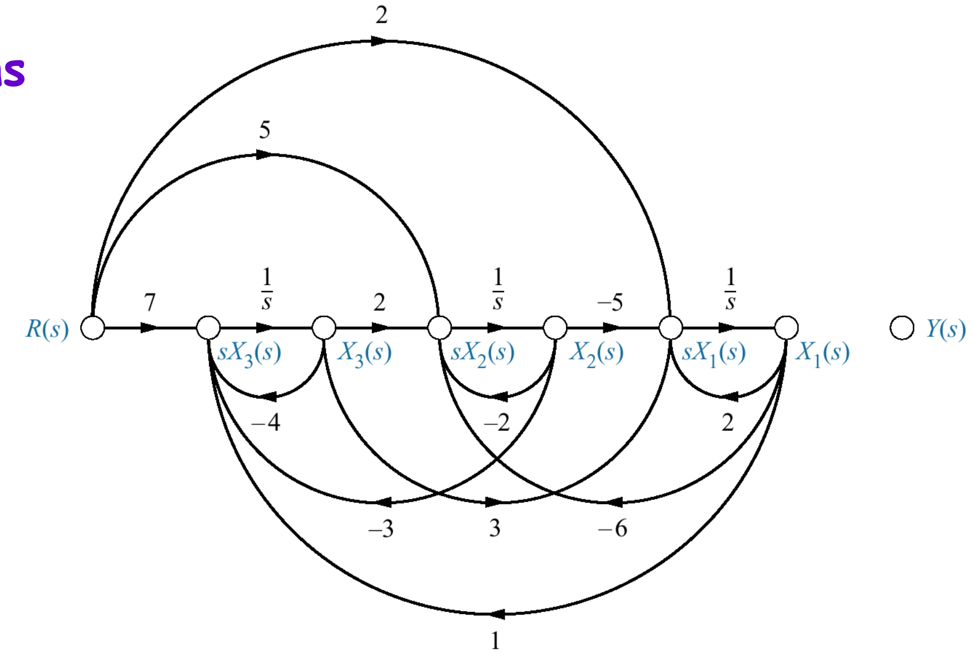


Signal-Flow Graphs of State Equations

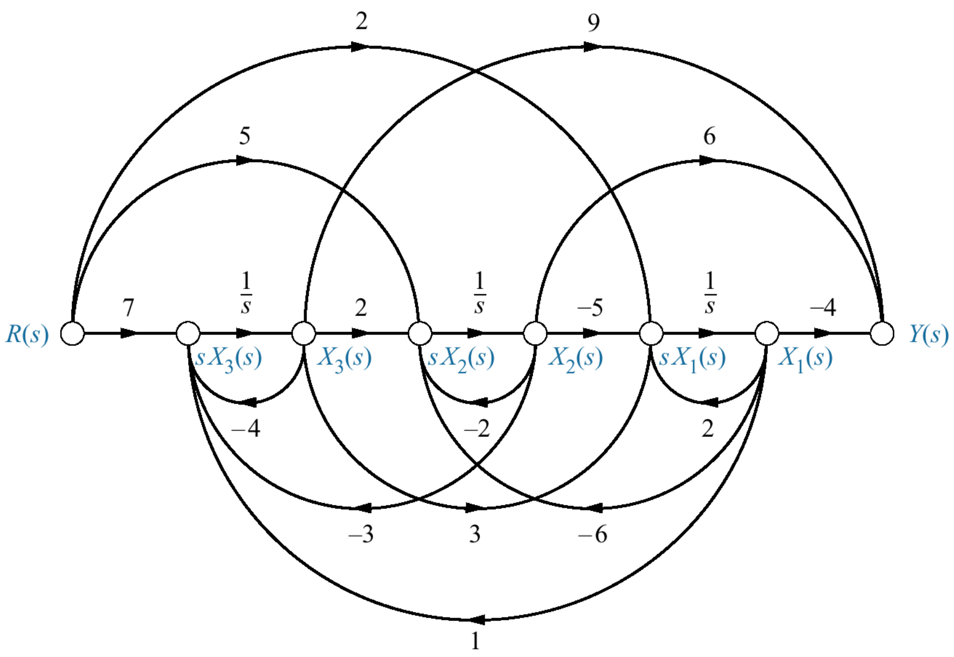
(continued)

e. form dx_3/dt;

f. form output



(e)



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Mathematical Models

1- *Classical* or *frequency-domain* technique

2- *State-Space* or *Modern* or *Time-Domain* technique

Classical or Frequency-Domain Technique

- Advantages

  - Converts differential equation into algebraic equation via transfer functions.

  - Rapidly provides stability & transient response info.

- Disadvantages

  - Applicable only to Linear, Time-Invariant (LTI) systems or their close approximations.

LTI limitation became a problem circa 1960 when space applications became important.

State-Space or Modern or Time-Domain Technique

- Advantages

  - Provides a unified method for modeling, analyzing, and designing a wide range of systems using matrix algebra.

  - Nonlinear, Time-Varying, Multivariable systems

- Disadvantages

  - Not as intuitive as classical method.

  - Calculations required before physical interpretation is apparent

State-Space Representation

An LTI system is represented in state-space format by the vector-matrix differential equation (DE) as:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t)$$ **Dynamic equation (s)**

$$\underline{y}(t) = C\underline{x}(t) + D\underline{u}(t)$$ **Measurement equations**

with $t \geq t_0$ *and initial conditions* $\underline{x}(t_0)$.

The vectors x, y, and u are the *state*, *output* and *input vectors*.
The matrices A, B, C, and D are the *system*, *input*, *output*, and *feedforward matrices*.

Definitions

System variables: Any variable that responds to an input or initial conditions.

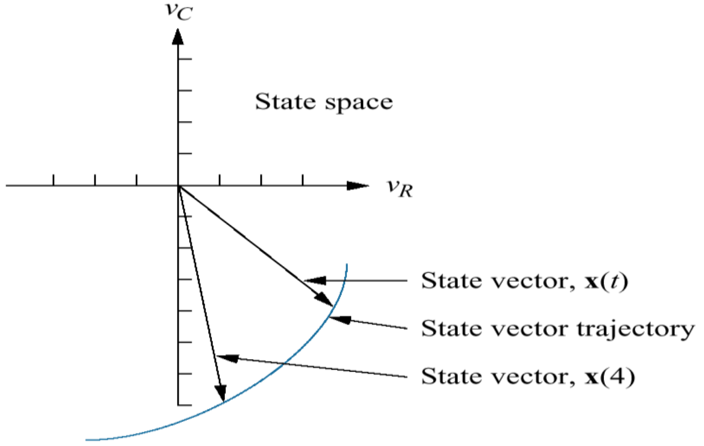
State variables: The smallest set of *linearly independent* system variables such that the initial condition set and applied inputs *completely determine the future behavior* of the set.

Linear Independence: A set of variables is linearly independent if none of the variables can be written as a linear combination of the others.

Definitions

State vector: An (n x 1) column vector whose elements are the state variables.

State space: The n-dimensional space whose axes are the state variables.



Graphic representation of state space and a state vector

The minimum number of state variables is equal to:

The order of the DE's describing the system.

The order of the denominator polynomial of its transfer function model.

The number of independent energy storage elements in the system.

Remember the state variables must be linearly independent! If not, you may not be able to solve for all the other system variables, or even write the state equations.

Dynamic equation

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t)$$ **State equation**

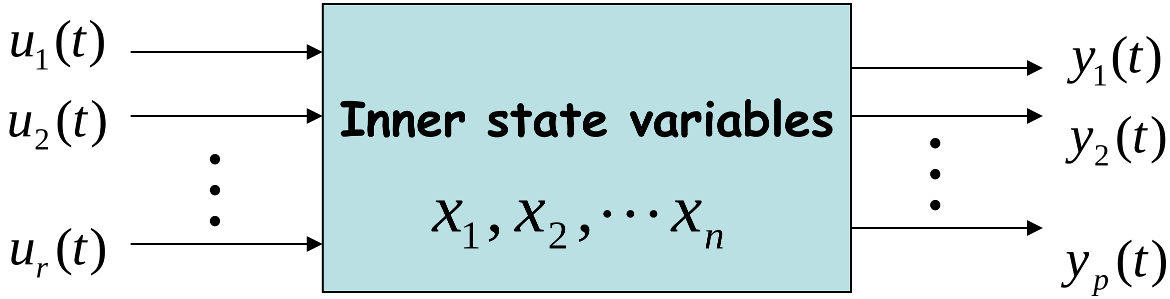
$$y(t) = Cx(t) + Du(t)$$ **Output equation**

State variable (→ $x_1(t)$)

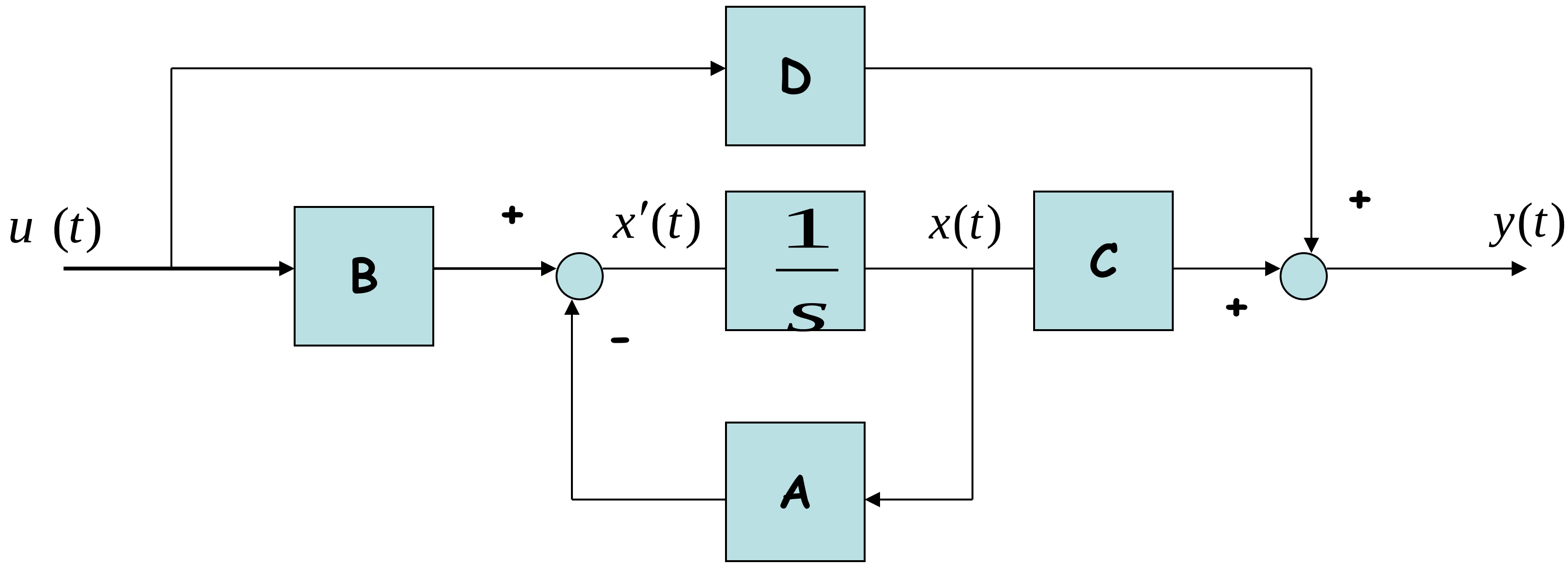
$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n\times 1} \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}_{r\times 1} \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}_{p\times 1} \quad x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix}_{n\times 1}$$

State space | **r- input** | **p- output**

$$A = \begin{bmatrix} & n\times n & \end{bmatrix} \quad B = \begin{bmatrix} & n\times r & \end{bmatrix} \quad C = \begin{bmatrix} & p\times n & \end{bmatrix} \quad D = \begin{bmatrix} & p\times r & \end{bmatrix}$$



$u_1(t)$
$u_2(t)$
$u_r(t)$
Inner state variables
$x_1, x_2, \cdots x_n$
$y_1(t)$
$y_2(t)$
$y_p(t)$



D
$u\ (t)$
B
+
$x'(t)$
$\frac{1}{s}$
$x(t)$
C
+
$y(t)$
-
+
A

Dynamical equation → **Transfer function**

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Laplace transform

$$sX(s) - x(0) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

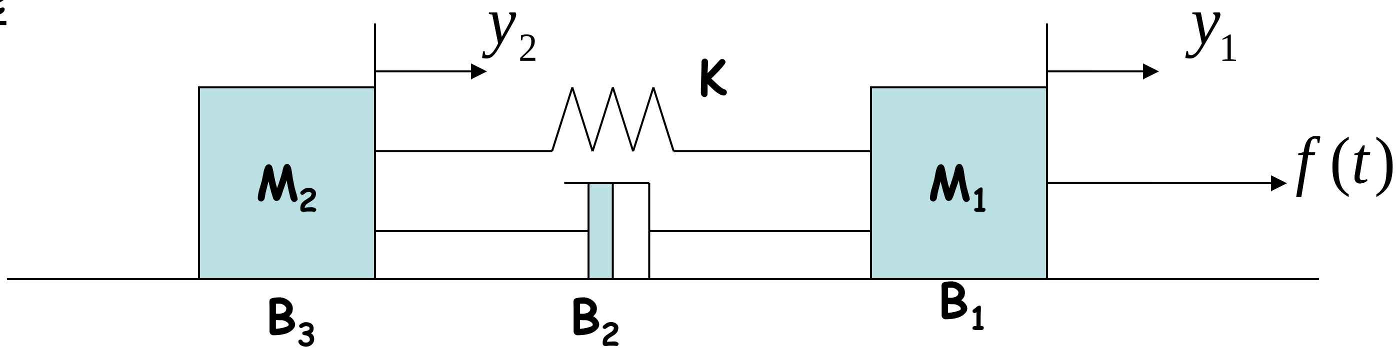
$$assume \quad x(0) = 0$$

$$X(s) = (sI - A)^{-1}BU(s)$$
$$Y(s) = [C(sI - A)^{-1}B + D]U(s)$$

matrix

Transfer function

Example

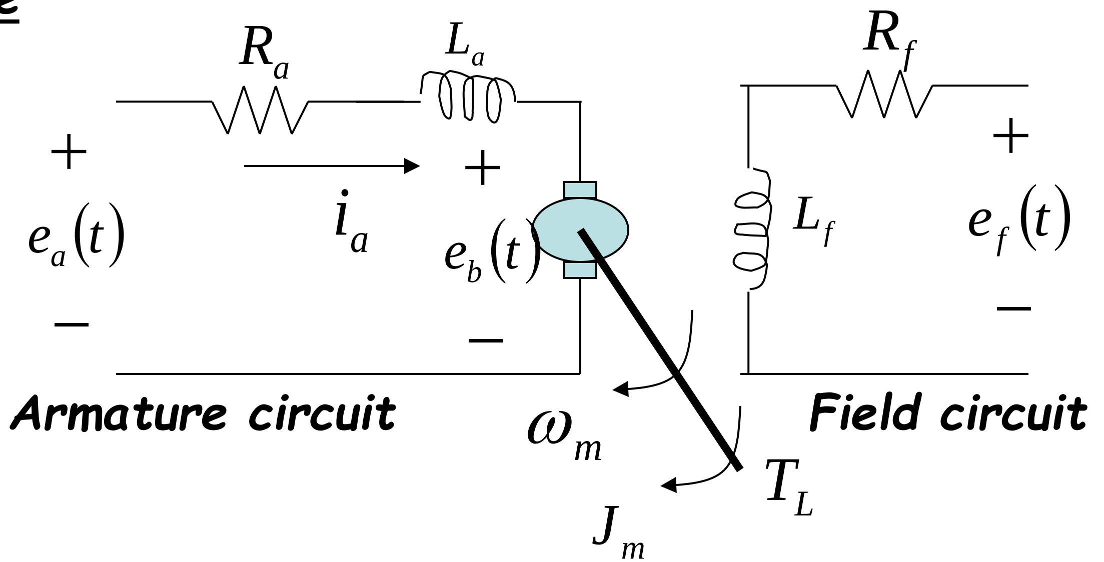


$$M_1\ddot{y}_1 + B_1\dot{y}_1 + B_2(\dot{y}_1 - \dot{y}_2) + K(y_1 - y_2) = f(t)$$

$$M_2\ddot{y}_2 + B_3\dot{y}_2 + B_2(\dot{y}_2 - \dot{y}_1) + K(y_2 - y_1) = 0$$

$$let \begin{cases} x_1 = y_1 - y_2 \\ x_2 = \dot{y}_1 \\ x_3 = \dot{y}_2 \end{cases} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K}{M_1} & -\frac{(B_1+B_2)}{M_1} & \frac{B_2}{M_1} \\ \frac{K}{M_2} & \frac{B_2}{M_2} & -\frac{(B_2+B_3)}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \end{bmatrix} f(t)$$

Example



$$\begin{cases} e_a(t) = R_a i_a + e_b + L_a \dfrac{di_a}{dt} \\ e_b = k_b \omega_m \end{cases}$$

$$\frac{di_a}{dt} = \frac{-R_a}{L_a} i_a - \frac{k_b}{L_a}\omega_m + \frac{1}{L_a} e_a$$

$$T_m = k_i i_a$$

$$= J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + T_L$$

$$\frac{d\omega_m}{dt} = \frac{k_i}{J_m} i_a - \frac{B_m}{J_m}\frac{d\theta_m}{dt} - \frac{1}{J_m} T_L$$

$$\frac{d\theta_m}{dt} = \omega_m$$

$$\begin{bmatrix} \dot{i}_a \\ \dot{\omega}_m \\ \dot{\theta}_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & \frac{-K_b}{L_a} & 0 \\ \frac{K_i}{J_m} & \frac{-B_m}{J_m} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J_m} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_a \\ T_L \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \theta_m(t) \\ \omega_m(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix}$$

With Our Best Wishes
Signals and Systems
Course Staff